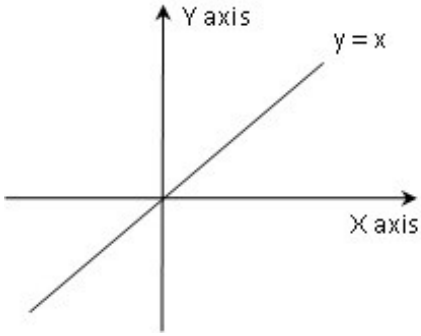
I have a dream… A dream that one day, I will see my students making a bonfire of all their pens and pencils… that I will see them lost in thought in my Quant class, occasionally drawing lines and curves on a drawing sheet with colorful crayons… I will see them coming up with innovative logical solutions, just like that… But I know that no dream of mine is realized until and unless I keep my nose to the grindstone (I am not waiting with bated breath to achieve that elusive target weight.) So on this particular sleepless night, I will write a post with some more figures, figures that make complicated questions look like easy pickings. Let me explain using step by step approach.

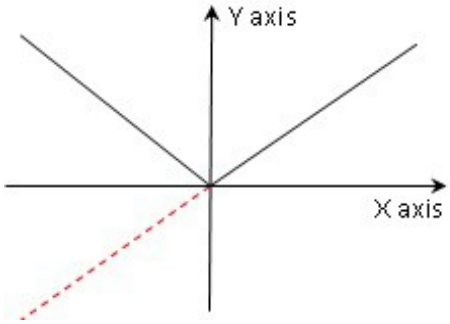
A Complicated Question: If y = ||x – 5| – 10|, for how many values of x is y = 1? (I remember once someone said, “I think I would rather eat spinach than try such questions.”)

Easy Pickings:

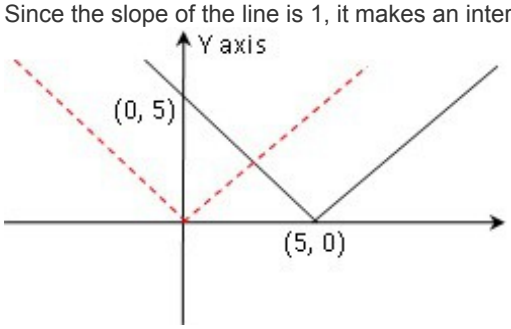
Graph of y = x is a line passing through the center with *slope 1.*



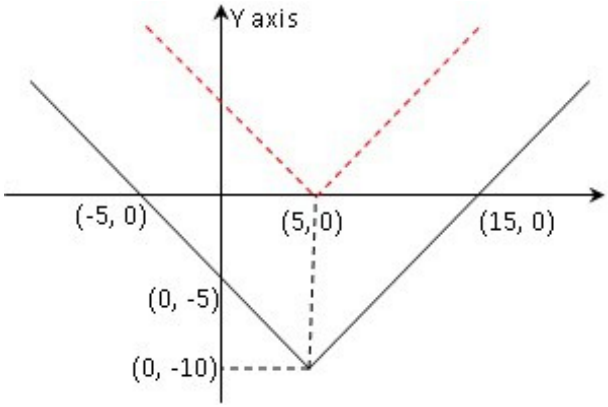
Graph of y = |x| is as shown below. Modulus cannot be negative so all negative values of y are flipped to positive. (The red line shows the original position for reference.)



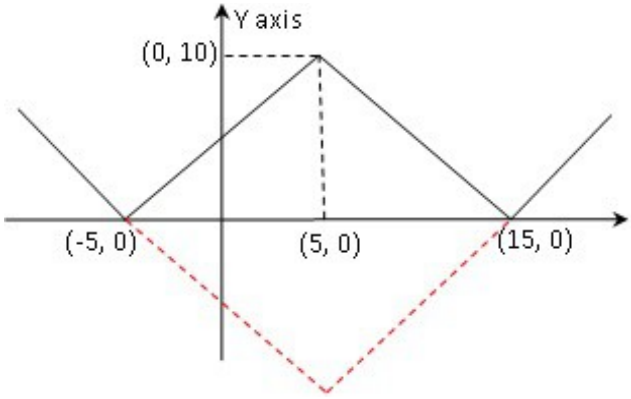
To get the graph of y = |x – 5|, shift the above graph 5 units to the right on the x axis. This is so because in the graph above, y = 0 when x = 0. But in the required graph, y should be 0 when x = 5. Hence the point at (0, 0) shifts to (5, 0). Since the slope of the line is 1, it makes an intercept of 5 on the y axis.



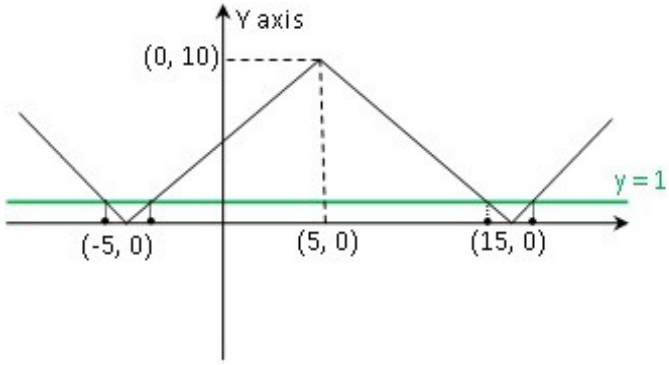
The graph of y = |x – 5| – 10 is just the above graph shifted down by 10 units because now y is 10 less than every previous value of y.



Now we need to take the modulus of the equation above to get y = ||x – 5| – 10|. Since a modulus is never negative, whatever part of the graph is below the x axis i.e. in quadrants III and IV, gets reflected above the x axis in quadrants I and II.



This is the graph we wanted. We see that the line y = 1 (shown in green below) intersects this graph at 4 points. So y = 1 when x = -6 or -4 or 14 or 16.



Put these values in the given equation if you want to cross check. Once you get the hang of it, you can arrive at this graph in under a minute! Such tricky questions can be elegantly handled using this approach. In fact, we can add many more levels of complexity and still easily arrive at our answer. For shakes, try out the graphs of y = |||x – 5| – 10| -5| and y = |||x| – 3| – x|!

Now that you have lost your sleep, I think I will sleep easier!